

SEMESTER LEARNING PLAN

**CALCULUS OF VARIATIONS COURSES
(23H01121803)**



TEACHING TEAM

STUDI PROGRAM OF MATHEMATICS - S1
FACULTY OF MATHEMATICS AND NATURAL SCIENCES
HASANUDDIN UNIVERSITY
MAKASSAR
2025

**STUDY PROGRAM OF MATEMATIKA - S1
FACULTY OF MATHEMATICS AND NATURAL SCIENCES
HASANUDDIN UNIVERSITY**

Vision

The scientific vision is to become a study program with an international reputation in the development of mathematics based on the Indonesian maritime continent by 2030

Vision Strategy

Mission

To fulfill the above vision, the Undergraduate Mathematics Study Program has four missions, namely:

- Organizing innovative and effective mathematics learning to improve the quality and creativity of students in order to compete nationally and internationally.
- Improving a research culture that produces internationally reputable publications.
- Playing an active role in community service activities and collaborating with other academic institutions, government, business, media and society.
- Carry out governance in the Mathematics Study Program that is effective, efficient and transparent based on IT and ISO 9001:2015 standards to achieve the tridharma goals.

Graduate Profiles

Gagal diterjemahkan

PLO charged to courses

CPL-1 (ILO 1) - Students are able to demonstrate an advanced understanding of basic pure and simple applied mathematics.

CPL-2 (P2) - The students are able to identify objects, techniques, and theorems in fundamental mathematics, and making a connection for solving problems

CPL-3 (KU1) - The students are able to analyse a mathematical problem with logic, analytic, and systematic structure

Course Learning Outcomes (CLO)

CPMK-1: Obtain extreme values in functional calculus; (CPL1)

CPMK-2: Reduce the Euler-Lagrange equation and find its relationship with extreme functions in the basic calculus; (CPL2)

CPMK-3: Determine the canonicity of the Euler equation and quantized function; (CPL3)

Sub-CLO

Sub CPMK-1: Introduction to Calculus of Variations Material. Identifying a functional, determining the maximum and minimum values (CPMK-1)

Sub CPMK-2: Applying first variations to the functional and extreme trajectories that optimize the functional for one or more functions (CPMK-1)

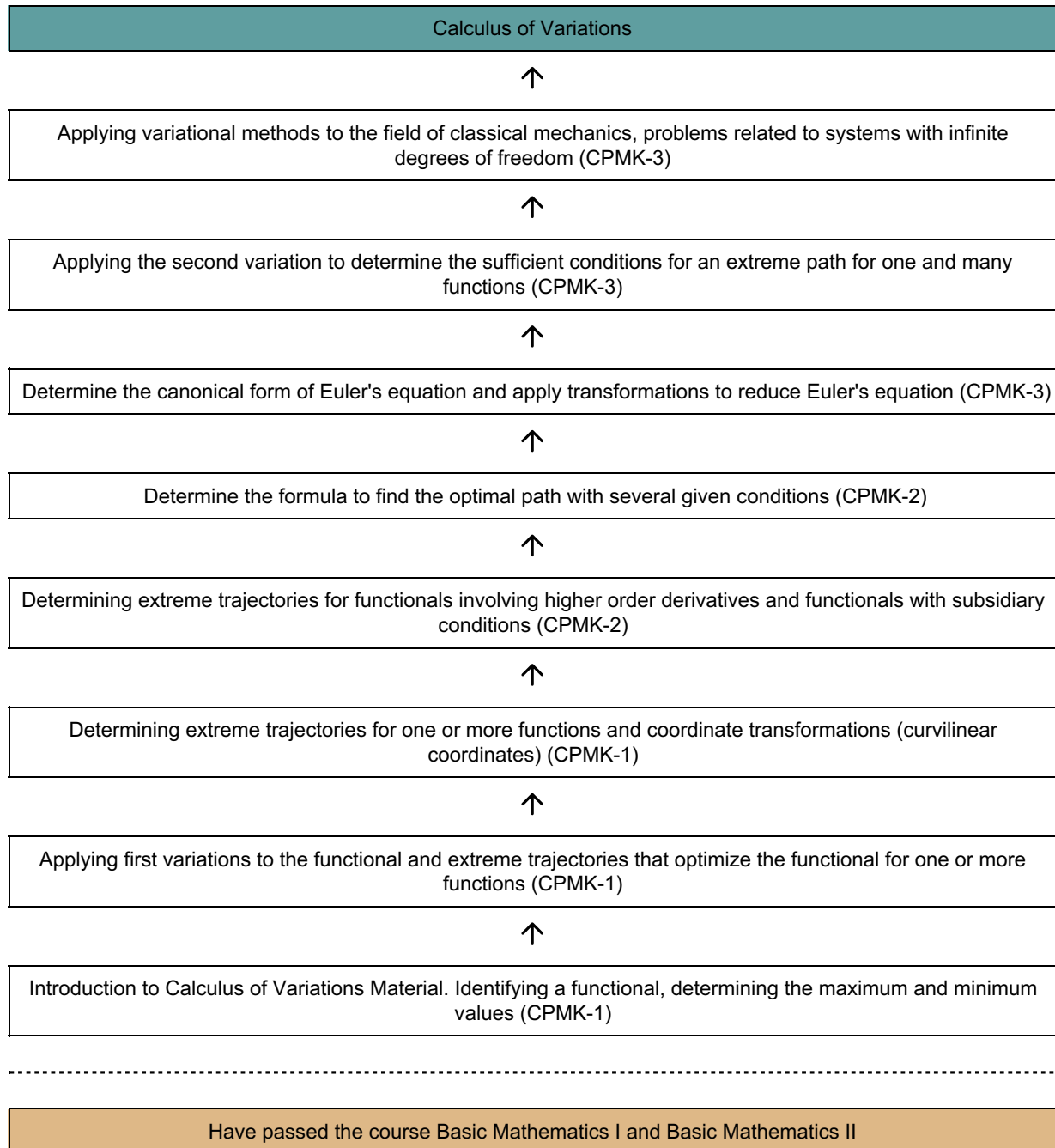
Sub CPMK-3: Determining extreme trajectories for one or more functions and coordinate transformations (curvilinear coordinates) (CPMK-1)

Sub CPMK-4: Determining extreme trajectories for functionals involving higher order derivatives and functionals with subsidiary conditions (CPMK-2)

Sub CPMK-5: Determine the formula to find the optimal path with several given conditions (CPMK-2)

- Sub CPMK-6: Determine the canonical form of Euler's equation and apply transformations to reduce Euler's equation (CPMK-3)
- Sub CPMK-7: Applying the second variation to determine the sufficient conditions for an extreme path for one and many functions (CPMK-3)
- Sub CPMK-8: Applying variational methods to the field of classical mechanics, problems related to systems with infinite degrees of freedom (CPMK-3)

Learning Analytics





HASANUDDIN UNIVERSITY

FAKULTY OF MATHEMATICS AND NATURAL SCIENCES

STUDY PROGRAM OF MATHEMATICS - S1

SEMESTER LEARNING PLAN

Course		Code	Course Group	Credits	SEMESTER	Compilation Date
Calculus of Variations		23H01121803	Elective Courses	3	4	10 Agustus 2025
AUTHORITY		SLP Developer Lecturer	Coordinator		Head of Study Program	
		Dr. Firman, S.Si.,M.Si., Prof. Dr. Eng. Mawardi, S.Si., M.Si.	Prof. Dr. Eng. Mawardi, S.Si., M.Si.		Dr. Firman, S.Si.,M.Si.	
Learning Outcomes Course	SLOs that are imposed on the course					
	SLO-1:	Mahasiwa memiliki pemahaman yang relatif mendalam dalam matematika murni dan matematika terapan sederhana.				
	SLO-2:	Mahasiswa mampu mengidentifikasi objek, teknik, dan sifat dalam matematika dasar, dan membuat koneksi untuk menyelesaikan masalah				
	SLO-3:	Mahasiswa mampu menganalisis suatu masalah matematika dengan logika, analitik, dan struktur sistematis				
	SLO ⇒ Course Learning Outcomes					
	After completing this course, it is expected:					
	SLO-1	CLO-1: Obtain extreme values in functional calculus;				
	SLO-2	CLO-2: Reduce the Euler-Lagrange equation and find its relationship with extreme functions in the basic calculus;				
	SLO-3	CLO-3: Determine the canonic of the Euler equation and quartzed function;				
	CLO ⇒ Sub-CLO					
	CLO-1	Sub-CLO-1:Introduction to Calculus of Variations Material. Identifying a functional, determining the maximum and minimum values				
		Sub-CLO-2:Applying first variations to the functional and extreme trajectories that optimize the functional for one or more functions				
		Sub-CLO-3:Determining extreme trajectories for one or more functions and coordinate transformations (curvilinear coordinates)				
	CLO-2	Sub-CLO-4:Determining extreme trajectories for functionals involving higher order derivatives and functionals with subsidiary conditions				
		Sub-CLO-5:Determine the formula to find the optimal path with several given conditions				
	CLO-3	Sub-CLO-6:Determine the canonical form of Euler's equation and apply transformations to reduce Euler's equation				
		Sub-CLO-7:Applying the second variation to determine the sufficient conditions for an extreme path for one and many functions				
		Sub-CLO-8:Applying variational methods to the field of classical mechanics, problems related to systems with infinite degrees of freedom				

Correlation between SLOs/CLOs to Sub-CLOs

SLOs that are charged on the Course	CPMK	SUB CPMK	Form of Assessment*					Weight	Value	Student Score
			Formative	Sumative						
				Independent Assignment	Case Studies	Written Exam	Written Exam			
SLO-1	CLO-1	SUB-CLO-1	discipline, perseverance, activeness	10	0	2.5	0	12.5		
SLO-1	CLO-1	SUB-CLO-2	discipline, perseverance, activeness	0	10	2.5	0	12.5		
SLO-1	CLO-1	SUB-CLO-3	discipline, perseverance, activeness	0	10	2.5	0	12.5		
SLO-2	CLO-2	SUB-CLO-4	discipline, perseverance, activeness	0	10	2.5	0	12.5		
SLO-2	CLO-2	SUB-CLO-5	discipline, perseverance, activeness	10	0	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-6	discipline, perseverance, activeness	0	10	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-7	discipline, perseverance, activeness	0	10	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-8	discipline, perseverance, activeness	0	10	0	2.5	12.5		
				20	60	10	10	100		
Course Description		This course introduces functional concepts, variational problems and function spaces. Next, the first and second variations are presented, the necessary conditions for an optimal trajectory, Euler's equations, and isoperimetric problems. Apart from that, the canonical form of Euler's equation, Legendre's transformation, Noether's theorem, Jacobi's theorem, and variational problems with double integrals are also presented. Through this course, students are expected to know functional problems and procedures for determining optimal trajectories with various conditions given. Apart from that, students are also expected to be able to determine the canonical form of Euler's equations and their transformations and apply them to various fields related to variational problems.								
Learning Materials/Subjects		1. Functional and Variational Problems (Functional and Variational Problems) 2. First variation and optimal path (First variation an optimal path) 3. Endpoint and derivative problems 4. Functional with higher order derivatives 5. Canonical form of Euler's equation 6. Quadratic functional (Quadratic functional) 7. Topics related to functional (Topics related to functional)								
		Main References								

Reference	1. I. M. Gelfand, S. V. Fomin, Calculus of Variations, Prentice-Hall, New Jersey, 1963. 2. Lewis, F.L. and Symros, V.L. Optimal Control. Canada: John Wiley and Sons, Inc., 1995. 3. Chiang, A.C. Elements of Dynamics Optimization. USA: Waveland Press, Inc., 1992.						
	Additional References						
	Gagal diterjemahkan						
Teaching Team							
Course requirement	Basic Mathematics I, Basic Mathematics II						
Week	Sub CPMK (End-of-stage learning ability)	Penilaian (Assesment)		Learning Forms and Methods [time estimate]		Content	Weight of Assessment (%)
		Indicator	Techniques & Criteria	Offline	Online		
1	2	3	4	5	6	7	8
1-2	Introduction to Calculus of Variations Material. Identifying a functional, determining the maximum and minimum values (CPMK-1)	Formative: accuracy of understanding Sumative: Clarity of a functional The accuracy of determining the maximum and minimum of a functional	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 01 Sumative Criteria: Independent Assignment (10) dinilai dengan rubrik 01 Assessment Technique: Gagal diterjemahkan	Studying: Collaborative learning (Collaborative Learning), other methods 2X3X50		Learning Contracts, An Overview of the Calculus of Variations. Functional, Variational Problems, and Function Spaces	10

3-4	Applying first variations to the functional and extreme trajectories that optimize the functional for one or more functions (CPMK-1)	Formative: accuracy of understanding Sumative: Accuracy of determining variations Accuracy of determining an extreme Clarity of interpretation of Euler's equation	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 01 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 01 Assessment Technique: Gagal diterjemahkan			The first variation of the Functional, the necessary conditions for an extreme path in the functional, and Euler's Equation	10
5-6	Determining extreme trajectories for one or more functions and coordinate transformations (curvilinear coordinates) (CPMK-1)	Formative: Accuracy of understanding Sumative: The accuracy of determining the optimal trajectory The accuracy of the transformation The accuracy of the parametric equations used - Teamwork	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 04 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 04 Assessment Technique: Gagal diterjemahkan			End points problems for one or more functions, Variational derivatives, and Invariant Euler Equations, Variations in parametric form	10

7	Determining extreme trajectories for functionals involving higher order derivatives and functionals with subsidiary conditions (CPMK-2)	Formative: accuracy of understanding Sumative: Optimal path determination accuracy Formulation accuracy problem The accuracy of determining a extreme	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 02 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 02 Assessment Technique: Gagal diterjemahkan			Functionals with higher order derivatives, isoperimetric problems and finite addition conditions	10
8	WRITTEN EXAMINATION						10
9	Determine the formula to find the optimal path with several given conditions (CPMK-2)	Formative: accuracy of understanding Sumative: Clarity of steps in determining the formula Accuracy in determining the conditions for determining the optimal trajectory	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 01 Sumative Criteria: Independent Assignment (10) dinilai dengan rubrik 01 Assessment Technique: Gagal diterjemahkan	Studying: Group discussion (Small Group Discussion), other methods 1X3X50		Derivation of the Basic Formula for the functional, including endpoints on the curve and Weierstrass-Erdmann conditions	10

10-12	Determine the canonical form of Euler's equation and apply transformations to reduce Euler's equation (CPMK-3)	Formative: accuracy of understanding Sumative: The accuracy of the canonical form of Euler's equation The accuracy of the transformation used	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 04 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 04 Assessment Technique: Gagal diterjemahkan	Studying: Group discussion (Small Group Discussion) 3X3X50	Canonical form of Euler's Equation, First Integral of Euler's Equation, Legendre Transformation, Noether's Theorem, Jaboci's Theorem.	10
13-14	Applying the second variation to determine the sufficient conditions for an extreme path for one and many functions (CPMK-3)	Formative: accuracy of understanding Sumative: The accuracy of determining extreme conditions The accuracy used in determining extremes	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 01 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 01 Assessment Technique: Gagal diterjemahkan	Studying: Group discussion (Small Group Discussion), other methods 2X3X50	Quadratic functional, Functional with many functions, Legendre's condition, Jacobi's necessary condition, sufficient condition for weak extremes.	10

15	Applying variational methods to the field of classical mechanics, problems related to systems with infinite degrees of freedom (CPMK-3)	Formative: accuracy of understanding Sumative: The accuracy of the method used The accuracy of selecting the canonical form The optimal trajectory accuracy obtained	Formative Criteria: discipline, perseverance, activeness dinilai dengan rubrik 02 Sumative Criteria: Case Studies (10) dinilai dengan rubrik 02 Assessment Technique: Gagal diterjemahkan	Studying: Group discussion (Small Group Discussion) 1X3X50		Canonical Forms of Euler's Equations and related topics. Variational problems involving double integrals.	10
16	WRITTEN EXAMINATION						10
							100

Matrix of SLO, CLO, and Assessment Method

SLO / CLO	CLO-1	CLO-2	CLO-3
CPL-1 (ILO 1)	Independent Assignment (Weight 10%) Case Studies (Weight 10%) Case Studies (Weight 10%)		
CPL-2 (P2)		Case Studies (Weight 10%) Independent Assignment (Weight 10%)	
CPL-3 (KU1)			Case Studies (Weight 10%) Case Studies (Weight 10%) Case Studies (Weight 10%)

Evaluation Type and Assessment Weight

Type	Assessment Weight
Independent Assignment	20
Case Studies	60
Written Exam	10
Written Exam	10
Total	100

Assessment and Evaluation of Student Achievement of CLOs

SLOs that are charged on the Course	CLO	SUB CLO	Form of Assessment*					Weight	Value	Student Score
			Formative	Sumative						
				Independent Assignment	Case Studies	Written Exam	Written Exam			
SLO-1	CLO-1	SUB-CLO-1	discipline, perseverance, activeness	10	0	2.5	0	12.5		
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SLO-2	CLO-2	SUB-CLO-4	discipline, perseverance, activeness	0	10	2.5	0	12.5		
SLO-2	CLO-2	SUB-CLO-5	discipline, perseverance, activeness	10	0	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-6	discipline, perseverance, activeness	0	10	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-7	discipline, perseverance, activeness	0	10	0	2.5	12.5		
SLO-3	CLO-3	SUB-CLO-8	discipline, perseverance, activeness	0	10	0	2.5	12.5		
				20	60	10	10	100		

Lampiran Rubrik 01 | ASSESMENT TERTULIS

Kriteria Penilaian	Bobot/Skor Penilaian				
	5	4	3	2	1/0
Konsep/ metode yang digunakan	Penjelasan konsep /metode (*) sangat lengkap dan akurat	Penjelasan konsep/metode (*) cukup jelas tetapi beberapa informasi tidak dituliskan secara lengkap.	Penjelasan konsep/metode (*) kurang jelas dan banyak informasi yang tidak dituliskan	Penjelasan yang dituliskan hampir tidak berkaitan dengan konsep/ metode (*)	Tidak memberikan konsep yang dibutuhkan
Sistematika penulisan/ pembuktian	Sistematika penulisan/ pembuktian sangat jelas dan terstruktur	Sistematika penulisan/ pembuktian cukup jelas namun ada langkah yang hilang	Sistematika penulisan/ pembuktian kurang jelas	Sistematika penulisan/ pembuktian tidak jelas	Jawaban tidak benar/ tidak ada
Interpretasi geometri/ kualitatif/ kuantitatif.	Interpretasi geometri/ kualitatif/ kuantitatif (*) tepat dan lengkap	Interpretasi geometri/ kualitatif/ kuantitatif (*) cukup lengkap/ tepat	Interpretasi geometri/ kualitatif/ kuantitatif (*) kurang lengkap/ tepat	Interpretasi geometri/ kualitatif/ kuantitatif (*) tidak lengkap/ tepat	Interpretasi geometri/ kualitatif/kuantitatif(*) tidak benar
Perhitungan/kesimpulan	Perhitungan/ kesimpulan sangat akurat/tepat dan disertai alasan yang mendasarinya	Perhitungan/ kesimpulan cukup akurat/tepat dan disertai alasan yang mendasarinya	Kesimpulan cukup tepat, namun tidak disertai alasan yang jelas	Perhitungan/ kesimpulan kurang akurat/tepat dan tidak disertai alasan yang mendasarinya	Perhitungan/kesimpulan salah

Lampiran Rubrik 02 | ASSESMENT PRESENTASI

Kriteria Penilaian	Bobot/Skor Penilaian				
	5	4	3	2	1
Penguasaan Materi	Mahasiswa sangat menguasai materi	Mahasiswa menguasai materi	Mahasiswa cukup menguasai materi	Mahasiswa kurang menguasai materi	Mahasiswa tidak menguasai materi
Sistematika Penyajian	Mahasiswa menyajikan materi presentasi dengan sangat sistematis	Mahasiswa menyajikan materi presentasi dengan sistematis	Mahasiswa menyajikan materi presentasi dengan cukup sistematis	Mahasiswa menyajikan materi presentasi dengan kurang sistematis	Mahasiswa menyajikan materi presentasi dengan tidak sistematis
Suara dan Ekspresi	Mahasiswa menjelaskan dengan suara yang sangat jelas, volume yang sangat sesuai, pengucapan istilah sangat tepat	Mahasiswa menjelaskan dengan suara yang jelas, volume yang sesuai, pengucapan istilah tepat	Mahasiswa menjelaskan dengan suara yang cukup jelas, volume yang cukup sesuai, pengucapan istilah cukup tepat	Mahasiswa menjelaskan dengan suara yang kurang jelas, volume yang kurang sesuai, pengucapan istilah kurang tepat	Mahasiswa menjelaskan dengan suara yang tidak jelas, volume yang tidak sesuai, pengucapan istilah tidak tepat
Kepercayaan Diri	Mahasiswa berbicara dengan sangat semangat, menularkan semangat dan antusiasme pada pendengar, eye contact ke semua peserta (audience)	Mahasiswa berbicara dengan semangat, menularkan semangat dan antusiasme pada pendengar, eye contact ke semua peserta (audience)	Mahasiswa berbicara dengan cukup semangat, cukup menularkan semangat dan antusiasme pada pendengar, eye contact ke semua peserta (audience)	Mahasiswa berbicara dengan kurang semangat, kurang menularkan semangat dan antusiasme pada pendengar, eye contact yang kurang ke semua peserta (audience)	Mahasiswa berbicara dengan tidak semangat, tidak dapat menularkan semangat dan antusiasme pada pendengar, tidak ada eye contact ke semua peserta (audience)
Kemampuan menjawab pertanyaan	Mahasiswa mampu menjawab semua pertanyaan dengan sangat tepat, sangat mendalam (lebih dari yang dibutuhkan), dan sangat tajam	Mahasiswa mampu menjawab semua pertanyaan dengan tepat, mendalam (lebih dari yang dibutuhkan), dan tajam	Mahasiswa mampu menjawab semua pertanyaan dengan cukup tepat, cukup mendalam, dan cukup tajam	Mahasiswa mampu menjawab semua pertanyaan dengan kurang tepat, kurang mendalam, dan kurang tajam	Mahasiswa mampu menjawab semua pertanyaan dengan tidak tepat, tidak mendalam, dan tidak tajam
Kepemimpinan (diskusi kelompok)	Mahasiswa sangat mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya presentasi	Mahasiswa mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya presentasi	Mahasiswa cukup mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya presentasi	Mahasiswa kurang mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya presentasi	Mahasiswa tidak mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya presentasi

Lampiran Rubrik 04 | ASSESMENT DISKUSI

Kriteria Penilaian	Bobot/Skor Penilaian				
	5	4	3	2	1
Kelugasan argumen	Mahasiswa menyampaikan pendapat menggunakan bahasa yang sangat baik, sangat mudah dimengerti, intonasi suara yang sangat jelas	Mahasiswa menyampaikan pendapat menggunakan bahasa yang baik, mudah dimengerti, intonasi suara yang jelas	Mahasiswa menyampaikan pendapat menggunakan bahasa yang cukup baik, cukup mudah dimengerti, intonasi suara yang cukup jelas	Mahasiswa menyampaikan pendapat menggunakan bahasa yang kurang baik, kurang mudah dimengerti, intonasi suara yang kurang jelas	Mahasiswa menyampaikan pendapat menggunakan bahasa yang tidak baik, tidak mudah dimengerti, intonasi suara yang tidak jelas
Parsipitasi aktif	Mahasiswa sangat aktif menyampaikan pendapat/masukannya, sangat aktif memberi jawaban, dan sangat berinisiatif mencari solusi	Mahasiswa aktif menyampaikan pendapat/masukannya, aktif memberi jawaban, dan berinisiatif mencari solusi	Mahasiswa cukup aktif menyampaikan pendapat/masukannya, cukup aktif memberi jawaban, dan cukup berinisiatif mencari solusi	Mahasiswa kurang aktif menyampaikan pendapat/masukannya, kurang aktif memberi jawaban, dan kurang berinisiatif mencari solusi	Mahasiswa tidak aktif menyampaikan pendapat/masukannya, tidak aktif memberi jawaban, dan tidak berinisiatif mencari solusi
Etika dan sikap tata nilai	Mahasiswa sangat mengindahkan etika diskusi akademik secara umum	Mahasiswa mengindahkan etika diskusi akademik secara umum	Mahasiswa cukup mengindahkan etika diskusi akademik secara umum	Mahasiswa kurang mengindahkan etika diskusi akademik secara umum	Mahasiswa tidak mengindahkan etika diskusi akademik secara umum
Kepemimpinan (diskusi kelompok)	Mahasiswa sangat mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya diskusi	Mahasiswa mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya diskusi	Mahasiswa cukup mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya diskusi	Mahasiswa kurang mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya diskusi	Mahasiswa tidak mampu menginisiasi, menggerakkan, mengarahkan, mengorganisir jalannya diskusi